Angles and segment measures are real numbers; therefore, we can do operations with them (add, subtract, multiply, divide) and apply equality properties to them. We can only do that, however, when we are using their ____________________ not the fact that they are congruent (i.e., not when we have a ____ sign).

Example 1: Can we apply operations to the following statements?

A) RT=PQ __________ B) $\overline{HI} \cong \overline{KU}$ _________ C) m∠ABC=m∠DEF __________ D) ∠A≅∠B=_______

Practice 1: Can we perform operations and/or apply equality properties to the following statements?

A) $\overline{QR} \cong \overline{ST}$ ________ B) m∠K > m∠T _________ C) PQ = ML __________ D) ∠CAT≅∠DOG _____

If we need to go between congruency and equality, though, we can use the **definition of congruency**, which states that If AB = XY, then ____________________.

Example 2: If we use the definition of congruency on the following statements, what would be the new statement?

A) YU=MQ _____________ B) m∠N = m∠H _________ C) $\overline{AM} \cong \overline{MB}$ _____________
D) ∠M≅∠Y=_______ E) ML> ST ____________________________

Practice 2: If we use the definition of congruency on the following statements, what would be the new statement?

A) $\overline{QR} \cong \overline{ST}$ ________ B) m∠K ≤ m∠T _________ C) PQ = ML __________
D) ∠CAT≅∠DOG _____ E) m∠K > m∠T _________

When we are just saying what something is, and it is not given, we use **definitions**. When performing operations and applying equality properties to angle and segment measures, we work mostly with definitions and postulates. Theorems are used when the segments and angles are already congruent. For example, if we are told that a point is the midpoint of a segment, and we are going to perform operations with the measures of those segments, then we would use the _____________________ of midpoint. If we are told that the point is the midpoint, and we just want to use the fact that each of the two halves are congruent, then we would use the midpoint _____________.

Here are some definitions and postulates that frequently appear in proofs dealing with measures:

Definition of a right angle: If an angle is a right angle, then its measure is _____.

Definition of complementary angles: If two angles are complementary, then their sum is ____.

Definition of supplementary angles: If two angles are supplementary, then their sum is ______.

Definition of perpendicular lines: If two lines are perpendicular, then they form ______ angles.

Definition of Congruency: If $\overline{QR} \cong \overline{ST}$, then ____________________

Definition of a Midpoint: If M is the midpoint of $\overline{AB}$, then _________________

Segment Addition Postulate: If K lies between J and L, then ____________________________

Definition of Congruency If AB = XY, then ____________________

Definition of Supplementary Angles: If ∠X and ∠Y are supplementary, then ______ + ______ = ____

Definition of Complementary Angles: If ∠X and ∠Y are complementary, then ______ + ______ = ____

Definition of a Right Angle: If ∠K is a right angle, then ________ = _____

Definition of Congruency: If ∠P ≅ ∠D, then _______ = _______

Angle Addition Postulate: If R is in the interior of ∠PQS, then _______ + _______ = _______

Overall, if you want to state that two angles or segments are congruent, you would use a theorem. If, after that, you need to perform operations with the angles or segments measures, then you would use the definition of congruency to go from congruence to measures. After that, you can apply the equality properties (i.e., addition, subtraction, multiplication, division, reflexive, symmetric, transitive, substitution, simplification)

Example 3: Complete the proof below:

Given: M is the midpoint of $\overline{AB}$ **Prove:** $\overline{AM} \cong \overline{MB}$

Statements	Reasons
1.	1.
2.	2.
3.	3.

Example 4: Complete the proof below:

Given: $\overline{JK} \cong \overline{QT}$; JK = 3x + 5; QT = 2x + 8

Prove: x = 3

Statement	Reason
1. $\overline{JK} \cong \overline{QT}$; JK = 3x + 5; QT = 2x + 8	1.
2. JK = QT	2.
3. 3x + 5 = 2x + 8	3.
4. x + 5 = 8	4.
5. x = 3	5.

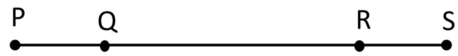
Practice 3: Given: M is the midpoint of $\overline{OG}$; OM = x + 4; MG = 5(3x-2)

Prove: x = 1

Draw a sketch:

Statement	Answers	Reason
1. M is the midpoint of $\overline{OG}$; OM = x + 4; MG = 5(3x-2)		A. Substitution
2. OM = MG		B. Subtraction Prop.
3. x + 4 = 5(3x-2)		C. Division Prop.
4. x + 4 = 15x – 10		D. Given
5. -14x + 4 = -10		E. Distributive Prop.
6. -14x = -14		F. Subtraction Prop.
7. x = 1		G. Definition of a midpoint

Practice 4: Given: PQ = RSProve: PR = QS



Statement	Reason
1. PQ = RS	1.
2. PQ + QR = RS + QR	2.
3. PQ + QR = PR RS + QR = QS	3.
4. PR = QS	4.

Example 5: Given: ∠O and ∠K are supplementary Prove: x = 25
m∠O = (4x + 10)°; m∠K = (3x – 5)°

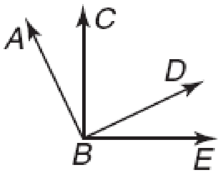
Statement	Reason
1. ∠O and ∠K are supplementary m∠O = (4x + 10)°; m∠K = (3x – 5)°	1.
2. m∠O + m∠K = 180°	2.
3. (4x + 10) + (3x – 5) = 180	3.
4. 7x + 5 = 180	4.
5. 7x = 175	5.
6. x = 25	6.

Practice 5: Given: R in the interior of ∠PQS;
m∠PQS = 70°; m∠PQR = (14x – 44)°; m∠RQS = 5x° Prove: x = 6

Sketch:

Statement	Answer	Reason
1. R in the interior of ∠PQS; m∠PQS = 70°; m∠PQR = (14x – 44)°; m∠RQS = 5x°		A. Substitution
2. m∠PQR + m∠RQS = m∠PQS		B. Simplify
3. (14x – 44) + 5x = 70		C. Division Prop.
4. 19x – 44 = 70		D. Given
5. 19x = 119		E. Addition Prop.
6. x = 6		F. Angle Addition Postulate

Practice 6: Given: ∠ABC and ∠CBD are complementary
∠DBE and ∠CBD form a right angle Prove: ∠ABC ≅ ∠DBE



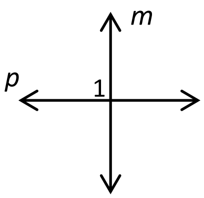
Statement	Reason
1.	1.
2. ∠DBE and ∠CBD are complementary	2.
3.	3.

Practice 7: Given: $\overrightarrow{AT}$ bisects ∠SAX;
m∠SAT = (6x – 4); m∠TAX = (2x + 28) Prove: x = 8

Sketch:

Statement	Reason
1.	1.
2.	2. Definition of an angle bisector
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Practice 8: Given: $p \perp m$ Prove: x = 16
m∠1 = (4x + 26)°



Statement	Reason
1. $p \perp m$	1. Given
2. ______ is a right angle	2.
3. m∠1 = ______	3.
4.	4.
5.	5.
6.	6.