

Recall the three properties of equality and congruence that we have done so far:

- 1) The _____ property says that a number or a segment, angle, etc. is equal to itself (i.e., $a=a$)
- 2) In the _____ property, the numbers, segments, angles, etc., switch places across the equal or the congruence sign. For example: $A=B$; therefore, $B=A$.
- 3) The _____ property requires at least two statements, and the elements in the center of the statements are the same. Those elements are eliminated, and we end up joining the first and the last parts of the statements.

Here is a list of **other properties of equality**:

Name of property	Definition	You may just write...	Explanation in words	Example
Addition Property of Equality	If $a = b$, then $a + c = b + c$.	Addition	You add the same amount on both sides of the equation.	
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.	Subtraction	You subtract the same number from both sides of the equation.	
Multiplication Property of Equality	If $a = b$, then $a * c = b * c$.	Multiplication	You multiply by the same number on both sides of the equation.	
Division Property of Equality	If $a = b$, then $a/c = b/c$.	Division	You divide by the same number on both sides of the equation.	
Substitution Property of Equality	If $a = b$, then you may replace b with a in any expression.	Substitution	You replace a variable, measure, angle, etc. by another one that is equivalent to it. The alternate value will take the place of the one being replaced.	
Commutative Property of Addition or Multiplication	$a + b = b + a$; $a * b = b * a$	Commutative property	The numbers being added or multiplied are moved around.	
Associative Property of Addition or multiplication	$(a + b) + c = a + (b + c)$; $(a * b) * c = a * (b * c)$	Associative property	The numbers or variables stay in the same place, but the parenthesis moves.	
Distributive Property	$a(b + c) = a*b + a*c$	Distributive property	The number outside the parenthesis is multiplied by all numbers inside the parenthesis.	
Identity property of addition or multiplication	$a + 0 = a$; $b*1 = b$	Identity property	You add 0 or multiply by 1 (which does not change the value of the original number)	
Inverse property of addition or multiplication	$a + (-a) = 0$; $b*(1/b) = 1$	Inverse property	You <i>add the opposite</i> to a number, and you end up with 0 (the additive identity), <i>or you multiply</i> a number by <i>its reciprocal</i> (where you flip a fraction), and you get 1 (the multiplicative identity).	
Multiplication property of zero	$a*0 = 0$	Zero property	Any number multiplied by 0 is equal to 0.	

Example 1: Write the letter of each property next to its definition.

- | | | | |
|-----|---|----|-------------------------------------|
| 1. | If $a = b$, then $b = a$ _____ | A. | Addition Property of Equality |
| 2. | If $a = b$, then $ac = bc$ _____ | B. | Subtraction Property of Equality |
| 3. | $\overline{AB} = \overline{AB}$ _____ | C. | Multiplication Property of Equality |
| 4. | $a = a$ _____ | D. | Division Property of Equality |
| 5. | If $a = b$, then $a + c = b + c$ _____ | E. | Reflexive Property of Equality |
| 6. | If $a(b + c) = ab + ac$ _____ | F. | Symmetric Property of Equality |
| 7. | If $a = b$ and $b = c$, then $a = c$ _____ | G. | Transitive Property of Equality |
| 8. | If $\angle P \cong \angle Q$, then $\angle Q \cong \angle P$ _____ | H. | Substitution Property of Equality |
| 9. | If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$ _____ | I. | Distributive Property |
| 10. | If $a = b$, and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$ _____ | J. | Reflexive Property of Congruence |
| 11. | If $a = b$, then b can be substituted for a _____ | K. | Symmetric Property of Congruence |
| 12. | If $a = b$, then $a - c = b - c$ _____ | L. | Transitive Property of Congruence |

Practice 1: Match the name to the definition

- | | |
|---|---|
| (1) <input type="checkbox"/> distributive property | (A) $a = a$ |
| (2) <input type="checkbox"/> reflexive property | (B) if $a = b$ then $ax = bx$ |
| (3) <input type="checkbox"/> commutative property of addition | (C) $ab = ba$ |
| (4) <input type="checkbox"/> multiplicative identity | (D) if $a = b$ then $a + x = b + x$ |
| (5) <input type="checkbox"/> multiplicative property of equality | (E) $a(b + c) = ab + ac$ |
| (6) <input type="checkbox"/> associative property of multiplication | (F) $x + 0 = x$ |
| (7) <input type="checkbox"/> associative property of addition | (G) $1x = x$ |
| (8) <input type="checkbox"/> commutative property of multiplication | (H) $a + b = b + a$ |
| (9) <input type="checkbox"/> additive property of equality | (I) $(-a)(-b) = ab$ |
| (10) <input type="checkbox"/> transitive property of equality | (J) if $a = b$ and $b = c$ then $a = c$ |
| (11) <input type="checkbox"/> additive identity | (K) $(ab)^n = a^n b^n$ |
| | (L) $(-a)b = a(-b) = -ab$ |
| | (M) $(ab)c = a(bc)$ |
| | (N) $(a + b) + c = a + (b + c)$ |

... for all a, b, c, x and n

Example 2: Identify the illustrated property:

- 1) if $x = 5$ then $3x = 3(5) = 15$ _____
- 2) if $a = b$ then $a - 8 = b - 8$ _____
- 3) if $a = b$ then $a(4) = b(4)$ _____
- 4) if $b = 5a$ and $b + 8 = 24$ then $5a$ can be substituted for b to get $5a + 8 = 24$ _____
- 5) if $a = b$ then $a + 3 = b + 3$ _____
- 6) if $a = b$ then $a/2 = b/2$ _____
- 7) if $x = a$ and $4a + 6 = 12$ then x can be substituted for a to get $4x + 6 = 12$ _____

Practice 2: Identify the illustrated property:

1. $x + y = y + x$
2. $6(m \cdot n) = (6 \cdot m)n$
3. $k + 0 = k$
4. $3t + 2r = 2r + 3t$
5. $6(u + 2v) = 6u + 12v$
6. $0 = 100 \cdot 0$
7. $(2a + 3b) + 4c = 2a + (3b + 4c)$
8. $pq + n = qp + n$
9. $gx = xg$
10. $15c + 15d = 15(c + d)$
11. $0 + b = b$
12. If $x + y = 3$, then $3 = x + y$
13. $x = x$
14. $4 \cdot 1 = 4$
15. $1 \cdot y = y$
16. $6 = 6$

Example 3: Write the name of the property that justifies the step to the left of the blanks given:

$$2x - 14 = x + 1$$

$$2x - 14 - x = x + 1 - x \quad \underline{\hspace{2cm}}$$

$$x - 14 = 1$$

$$x - 14 + 14 = 1 + 14 \quad \underline{\hspace{2cm}}$$

$$x = 15$$

Practice 3: Write the name of the property that justifies the step to the left of the blanks given:

$$f + 4 = -6$$

$$(f + 4) - 4 = (-6) - 4$$

$$f + 4 - 4 = -6 - 4 \quad \underline{\hspace{2cm}}$$

$$f + 0 = -10$$

A) $f = -10$

$$x + 12 = 5$$

$$x + 12 - 12 = 5 - 12 \quad \underline{\hspace{2cm}}$$

B) $x = -7$

$$3y - 12 = 0$$

$$3y - 12 + 12 = 0 + 12 \quad \underline{\hspace{2cm}}$$

$$3y = 12$$

$$\frac{3y}{3} = \frac{12}{3} \quad \underline{\hspace{2cm}}$$

C) $y = 4$

$$2w + 12 = 40$$

$$2w + 12 - 12 = 40 - 12 \quad \underline{\hspace{2cm}}$$

$$2w = 28$$

$$2w \div 2 = 28 \div 2 \quad \underline{\hspace{2cm}}$$

D) $w = 14$

$$ab(a+b) = (ab)a + (ab)b \quad \underline{\hspace{2cm}}$$

$$= a(ab) + (ab)b$$

$$= (a \cdot a)b + a(b \cdot b)$$

$$= a^2b + ab^2$$

E)