Two-column proofs

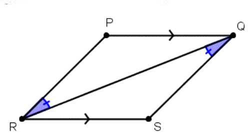
A two-column proof has numbered statements and reasons that show the logical order of the argument. Each statement has a reason listed to its right. We will be using mostly two-column proofs, as they tend to be the most commonly-used proofs in Geometry.

How to write a proof:

- Number each step.
- Start with the given information.
- Statements with the same reason can be combined into one step. It is up to you.
- Draw a picture (if not given one) and mark it with the given information.
- You must have a reason for EVERY statement.
- Reasons could be definitions, postulates, properties, theorems and corollaries. "Given" is only used as a reason if the information in the statement column was told in the problem or shown in the picture.
- The order of the statements in the proof is not always fixed, but make sure the order makes logical sense. If a statement relies on another statement, list it later than the statement it relies on.
- Use symbols and abbreviations for words within proofs. For example, ≅ can be used in place of the word *congruent*. You could also use ∠ for the word *angle*.
- End the proof with the statement you are trying to prove.

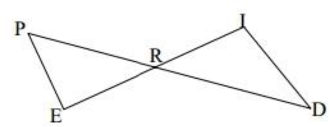
Examples 1 and 2:

Given : $\overline{PQ} \parallel \overline{RS}$
$\angle PRQ \cong \angle SQR$
Prove : $\triangle PQR \cong \triangle SRQ$



Statements	Reasons
1. $\overline{PQ} \parallel \overline{RS}$	1. Given
2. $\angle PRQ \cong \angle SQR$	2. Given
3. $\angle PQR \cong \angle SRQ$	3. Alternate Interior Angles Theorem
4. $\overline{RQ} \cong \overline{RQ}$	4. Reflexive Property
5. $\triangle PQR \cong \triangle SRQ$	5. ASA Postulate

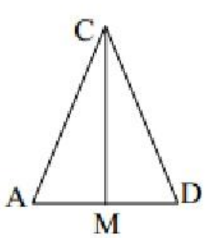
Given: R is the midpoint of $\overline{PD}$ and $\overline{EI}$ Prove: ΔPRE ≅ ΔDRI



Statement	Reason
1. R is the midpoint of $\overline{PD}$ and $\overline{EI}$	1.
2.	2.
3.	3.
4. ΔPRE ≅ ΔDRI	4.

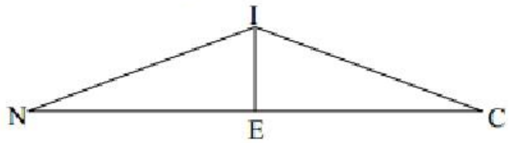
Practice: Complete the two-column proofs below

A) Given: $\overline{AC} \cong \overline{DC}$, M is the midpoint of $\overline{AD}$ Prove: ΔACM ≅ ΔDCM



Statement	Reason
1. $\overline{AC} \cong \overline{DC}$, M is the midpoint of $\overline{AD}$	1.
2. $\overline{AM} \cong \overline{MD}$	2.
3. $\overline{CM} \cong \overline{CM}$	3.
4. ΔACM ≅ ΔDCM	4.

Given: $\overline{IE} \perp \overline{NC}$, E is the midpoint of $\overline{NC}$ Prove: ΔNIE ≅ ΔCIE



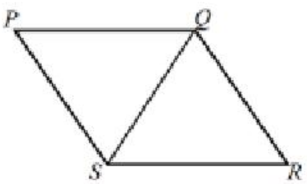
B)

Statement	Reason
1. E is the midpoint of $\overline{NC}$	1.
2.	2.
3.	3.
4. $\overline{IE} \perp \overline{NC}$	4.
5.	5.
6.	6.
7. ΔNIE ≅ ΔCIE	7.

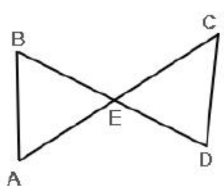
Example 3: Write a two-column proof

Given: $\overline{PQ} \parallel \overline{SR}$

Prove: ΔPQS≅ΔRQS

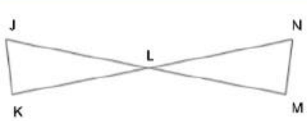


Practice: Write a two-column proof



C) Given: E is the midpoint of segments BD and AC

Prove: ΔABE≅ΔDCE



D) JM bisects KN, KN bisects JM, JK≅ NM

Prove: ΔJLK≅ΔNLM

In real life, there are times when we have to defend our point of view of prove that what we are saying is true. The same occurs in Geometry. In Geometry, we are often asked to prove a certain point using information that we are given, and then use properties, definitions, etc., to come to the conclusion that we want. For that, we have to use reasoning.

Some key concepts on reasoning and proofs:

- A __________________ is a good guess or an idea about a pattern.

- A __________________ or axiom is a statement that is accepted as true (e.g., through any two points, there is exactly one line; if two lines intersect, then their intersection is exactly one point).

- Undefined terms, definitions, postulates and algebraic properties are used to prove that other statements or conjectures are true. Once a statement or conjecture has been shown to be true, it is called a ______________, and it can be used to justify that other statements are true.

- A conditional statement is a statement that can be written in the form "if *p*, then *q*". The phrase immediately following the word "if" is called the ________________, and the phrase immediately following the word "then" is called the ________________. For example: If you buy a car, then you get $1,500 cash back. In this example, the hypothesis is "you buy a car", and the conclusion is "you get $1,500 cash back".

- There are two types of reasoning: inductive and deductive reasoning. __________________ _____________________ uses examples to make a conjecture. _________________ _______________ uses facts, rules, definitions, or properties to reach logical conclusions. Inductive reasoning by itself does not prove anything, but deductive reasoning can be used to prove statements.

- A ___________ is a convincing argument that shows why a statement is true. In a proof, each statement you make is supported by a statement that is accepted as true. One type of proof is called a ________________ _______________ or informal proof. In this type of proof, you write a paragraph to explain why a conjecture for a given situation is true.

There are different kinds of proofs. We have paragraph proofs, flow chart proofs, coordinate proofs, indirect proof, and two-column proofs. Here are some examples:

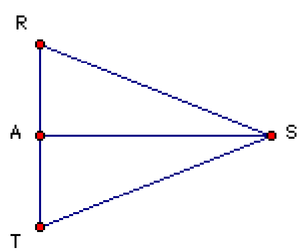
In a **paragraph proof**, we write a paragraph that organizes our thinking process in a logical way.

If $\overline{AS} \perp \overline{RT}$,

and A is a midpoint of $\overleftrightarrow{RT}$

prove

$\triangle RAS \cong \triangle TAS$



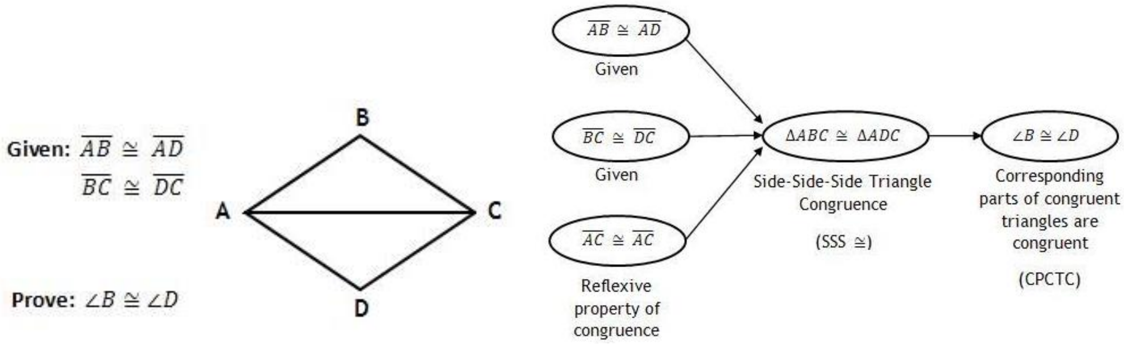
Since A is a midpoint, the segments $\overline{RA}$ and $\overline{AT}$ are congruent by definition of midpoint.

Since $\overline{AS}$ and $\overline{RT}$ are perpendicular, then < RAT and < TAS are right angles and congruent to each other.

Last, $\overline{AS}$ is congruent to itself by reflexive property.

So $\triangle RAS \cong \triangle TAS$ by SAS.

Flow charts or proofs are one of the ways that theorems can be proved. A flow chart in geometry is much like the one used in other classes or in business. It is a visual way to organize your reasoning. Arrows connect statements that help to prove the theorem. Reasons are written below the statements. Reasons can be properties, definitions, postulates, or already proven theorems.

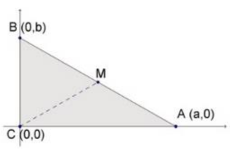


A **coordinate proof** uses points on an x-y coordinate plane and algebra to prove what we are trying to prove.

__Prove:__ The midpoint of the hypotenuse of a right triangle is equidistant from the three vertices.

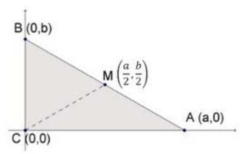
__Solution:__

- Step 1: First we make a coordinate diagram of the triangle and note what we are given and what we must prove.
  - Given: $\angle C$ is a right angle. M is the midpoint of $\overline{AB}$.
  - Prove: $MC = MA$.
  (We already know that $MB = MA$.)



- Step 2: Next we use what is given to add information to the diagram or to express algebraically any given fact not shown in the original diagram.

(In this example, we use the given fact that M is the midpoint of $\overline{AB}$ to find the coordinates of M.)



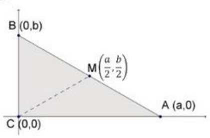
- Step 3: Finally, we reword what we are trying to prove in algebraic terms. To prove $MC = MA$:

$$MC = \sqrt{\left(\frac{a}{2}-0\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$MC = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$MA = \sqrt{\left(\frac{a}{2}-a\right)^2 + \left(\frac{b}{2}-0\right)^2} = \sqrt{\left(-\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$MA = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$



$\therefore MC = MA.$
Since $MA = MB$,
$MA = MB = MC.$

An **indirect proof**, also known as proof by contradiction, is when you start by assuming that what you are trying to prove is false. Then, if that assumption leads to a contradiction, then what you are trying to prove is true.

Example: Prove $\sqrt{2}$ is irrational.

Proof: Assume $\sqrt{2}$ is rational. That means that $\sqrt{2} = \frac{a}{b}$, where a and b are relatively prime integers.

$$\sqrt{2} = \frac{a}{b}$$

$$2 = \frac{a^2}{b^2}$$

$$2b^2 = a^2$$

Since $2b^2 = a^2$, 2 must be a factor of a^2. 2 is thus a factor of a, so it turns out that 4 is a factor of a^2. Since 4 is a factor of a^2 and $2b^2 = a^2$, it follows that 4 is a factor of $2b^2$. Hence 2 must be a factor of b^2. This means that 2 must be a factor of b. 2 is thus a factor of both a and b, so a and b are not relatively prime. This contradicts the assumption that $\sqrt{2}$ is rational. By contradiction, $\sqrt{2}$ is irrational.