Objective: To simplify square roots

Review:
A perfect _______________ is a number that results after multiplying one number by itself.

Complete the following table of perfect squares:

$1^2=$	$6^2=$	$11^2=$	$16^2=$
$2^2=$	$7^2=$	$12^2=$	$17^2=$
$3^2=$	$8^2=$	$13^2=$	$18^2=$
$4^2=$	$9^2=$	$14^2=$	$19^2=$
$5^2=$	$10^2=$	$15^2=$	$20^2=$

The opposite of a square is a ________________________________.

Example and practice 1: Find the square root of the following numbers:

A) 400 C) 25 E) 256

B) 121 D) 169 F) 196

There are times when we are trying to find the square root of numbers that are not perfect squares. In that case, we can just use the calculator to get an approximate decimal. However, if we are trying to find an exact number, we can simplify the square root.

There are two methods for simplifying square roots. The **<u>first method</u>** is:
1) Find the largest perfect square that goes into the number inside the radical. If you cannot identify the largest perfect square root right away, you can divide the number inside the radical by perfect squares using a calculator until you get a whole number.
2) Split the number inside the radical into the perfect square and the number by which you would have to multiply it in order to get the number inside the radical.
3) Square root the perfect square and put it outside the new radical.
4) Place the other number inside the new radical.

Note: If the number inside the radical can still be simplified, you will need to repeat the process until the number inside the radical cannot be simplified.

Example 2: Simplify the following square roots by finding the perfect square:

A) $\sqrt{45}$ B) $\sqrt{1000}$ C) $\sqrt{18}$

Practice 2: Simplify the following square roots by finding the perfect square:

A) $\sqrt{96}$ B) $\sqrt{250}$ C) $\sqrt{175}$

D) $\sqrt{216}$ E) $\sqrt{72}$ F) $\sqrt{343}$

The **<u>second method</u>** is:

1) Find the prime factorization of the number inside the square root.
2) Group the prime numbers that are the same in pairs.
3) Multiply the **pairs** of prime numbers counting each pair as one factor. The product becomes the number **outside** the new radical.
4) Multiply the numbers with **no pairs**. Their product becomes the number **inside** the new radical.

Example 3: Simplify the following square roots by finding their prime factorization:

A) $\sqrt{200}$ B) $\sqrt{80}$ C) $\sqrt{12}$

Practice 3: Simplify the following square roots by finding their prime factorization:

A) $\sqrt{32}$ B) $\sqrt{75}$ C) $\sqrt{54}$

D) $\sqrt{48}$ E) $\sqrt{24}$ F) $\sqrt{98}$

Objective: To simplify radical expressions involving multiplication of square roots

Whenever we multiply radical expressions, we multiply the numbers outside the radicals (called "coefficients") with the numbers ___________________ the radicals; and the numbers inside the radicals (called "radicands") with the numbers ___________________ the radicals. We have to reduce our radicals either in the beginning or the end, if needed.

Example 1: Simplify the following expressions:

A) $\sqrt{12} \cdot \sqrt{2}$ B) $\sqrt{8} \cdot \sqrt{18}$ C) $\sqrt{40} \cdot \sqrt{5}$

Practice 1: Simplify the following expressions:

A) $\sqrt{15} \cdot \sqrt{3}$ B) $\sqrt{8} \cdot \sqrt{6}$ C) $\sqrt{50} \cdot \sqrt{2}$

Example 2: Simplify the following expressions:

A) $5\sqrt{7} \cdot \sqrt{20}$ B) $\sqrt{32} \cdot 3\sqrt{2}$ C) $4\sqrt{40} \cdot \sqrt{3}$

Practice 2: Simplify the following expressions:

A) $\sqrt{24} \cdot 8\sqrt{2}$ B) $9\sqrt{7} \cdot \sqrt{3}$ C) $\sqrt{5} \cdot 7\sqrt{5}$

Example 3: Simplify the following expressions:

A) $2\sqrt{10} \cdot 3\sqrt{10}$

B) $4\sqrt{12} \cdot 3\sqrt{6}$

C) $8\sqrt{3} \cdot 7\sqrt{5}$

Practice 3: Simplify the following expressions:

A) $3\sqrt{27} \cdot 9\sqrt{2}$

B) $2\sqrt{7} \cdot 5\sqrt{7}$

C) $10\sqrt{11} \cdot 12\sqrt{13}$

Example 4: Simplify the following expressions:

A) $3\sqrt{12} \cdot \sqrt{6}$

B) $\sqrt{5} \cdot \sqrt{10}$

C) $4\sqrt{48} \cdot 6\sqrt{20}$

Practice 4: Simplify the following expressions:

A) $\sqrt{5} \cdot -4\sqrt{20}$

B) $\sqrt{2}(\sqrt{6})$

C) $2\sqrt{32} \cdot -6\sqrt{45}$

Squaring a number is the same as multiplying it by itself. Likewise, when we square a radical expression, we multiply that radical expression by itself, and follow one of the processes above. There is a shortcut, though. Whenever we square a radical, we can "___________________" the exponent, and then simplify. Remember that squares and square roots are opposites; so, if we square a square root, the radical disappears.

Example 5: Simplify the following expressions:

A) $(4\sqrt{6})^2$ B) $(5\sqrt{7})^2$ C) $(7\sqrt{2})^2$

Practice 5: Simplify the following expressions:

A) $(3\sqrt{2})^2$ B) $(15\sqrt{5})^2$ C) $(24\sqrt{3})^2$

If we have variables, we treat the variables like just another factor.

Example 6: Simplify the following expressions:

A) $-\sqrt{32d} \cdot \sqrt{63d}$ B) $\sqrt{20x^2} \cdot \sqrt{20x}$

Practice 6: Simplify the following expressions:

A) $2\sqrt{20r} \cdot 5\sqrt{99r}$ B) $\sqrt{18a^2} \cdot 4\sqrt{3a^2}$

Objective: To simplify radicals in a fraction

Recall that whenever we have a radical, we can split the number inside the radical (the radicand) into two factors, and each factor can have its own radical. For example: $\sqrt{28} = \sqrt{4} \cdot \sqrt{7}$ since 28=4 ·7. Likewise, whenever we have a fraction inside a radical, we can separate the numerator and the denominator into its own radical; therefore, we can find the square root of each one of those numbers separately. At the end, we reduce the fraction with which we end up, if necessary.

Example 1: Find the square root of the following fractions:

A) $\sqrt{\frac{9}{100}}$ B) $\sqrt{\frac{64}{100}}$

C) $\sqrt{\frac{144}{225}}$ D) $\sqrt{\frac{16}{36}}$

Practice 1: Find the square root of the following fractions:

A) $\sqrt{\frac{4}{25}}$ B) $\sqrt{\frac{1}{49}}$

C) $\sqrt{\frac{16}{144}}$ D) $\sqrt{\frac{9}{81}}$

Whenever we simplify a fraction with radicals, radicals are not allowed to be in the denominator at the end; therefore, we will have to ________________________ the radical. Rationalizing the radical refers to getting rid of the radical in the denominator and turning it into a rational number, which could be either a whole number or a decimal. What we do is based on the __________________ property.

Recall that when we multiply a number by 1, its value does not change. Recall also that a fraction where both the numerator and the denominator are the same is equal to ___ because such fraction is a division of both numbers since a fraction is a division.

Therefore, in order to rationalize the radical where the denominator has a square root, we have to multiply the fraction by a fraction where both the numerator and the denominator are the denominator of the original radical.

So, the **process** for dividing radicals (or simplifying a fraction with radicals in the denominator) is to
1) Simplify the radicals, if possible
2) Rationalize the fraction
3) Multiply across (outside with outside and inside with inside)
4) Reduce the new fraction

Note: You can simplify either before, during, or after the division process.

Example 2: Simplify the fraction

A) $\frac{\sqrt{10}}{\sqrt{32}}$

B) $\dfrac{\sqrt{6}}{\sqrt{75}}$

C) $\dfrac{\sqrt{12}}{\sqrt{27}}$

D) $\dfrac{\sqrt{18}}{\sqrt{64}}$

Practice 2: Simplify the fraction

A) $\dfrac{\sqrt{27}}{\sqrt{75}}$

B) $\dfrac{\sqrt{14}}{\sqrt{12}}$

C) $\dfrac{\sqrt{8}}{\sqrt{25}}$

D) $\dfrac{\sqrt{32}}{\sqrt{50}}$

There will be times when we have a coefficient outside the radical. If that is the case, we have the option of reducing the coefficients if possible before we do the rest of the process. Notice that is it optional as you can simplify before, during or after the division process.

Example 3: Simplify the fraction

A) $\dfrac{5\sqrt{32}}{10\sqrt{20}}$

B) $\dfrac{10\sqrt{8}}{15\sqrt{14}}$

C) $\dfrac{\sqrt{27}}{12\sqrt{8}}$

Practice 3: Simplify the fraction

A) $\dfrac{24\sqrt{40}}{12\sqrt{46}}$

B) $\dfrac{3\sqrt{24}}{9\sqrt{45}}$

C) $\dfrac{5\sqrt{128}}{\sqrt{75}}$

Sometimes, if we start with a fraction where both the numerator and denominator are radicals, we can turn it into a radical with a fraction inside, reduce the fraction, and the square root the numbers separately; but that is not necessary, you can still just rationalize the radical and simplify.

Example 4: Simplify the fraction

A) $\dfrac{\sqrt{2}}{\sqrt{18}}$

B) $\dfrac{\sqrt{8}}{\sqrt{4}}$

Practice 4: Simplify the fraction

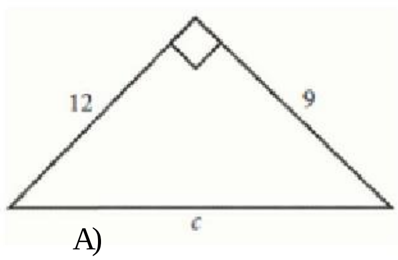
A) $\dfrac{\sqrt{5}}{\sqrt{20}}$

B) $\dfrac{\sqrt{3}}{\sqrt{48}}$

Objective: To discover the ratio of special right triangles

Review:
The Pythagorean Theorem helps us to find the length of a leg or the hypotenuse of a _______ triangle, given the other two measures. The formula for the theorem is ______________________. Remember that the legs could be either ____ or _____, but the hypotenuse is always _____.

Example 1: Find the missing length using the Pythagorean Theorem.



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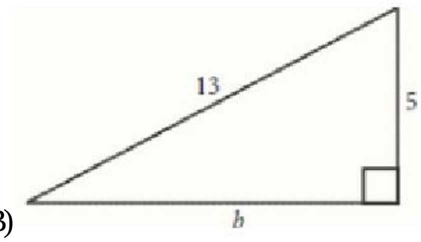

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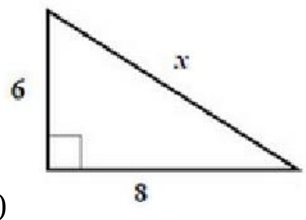
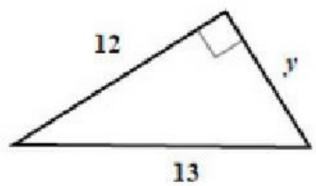
Practice 1: Find the missing length using the Pythagorean Theorem.



A)



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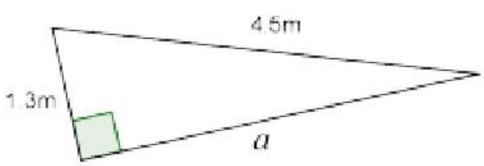
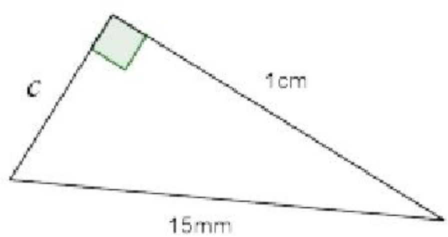
The Pythagorean Theorem works even when the lengths are not whole numbers.

Example 2: Find the missing length using the Pythagorean Theorem.

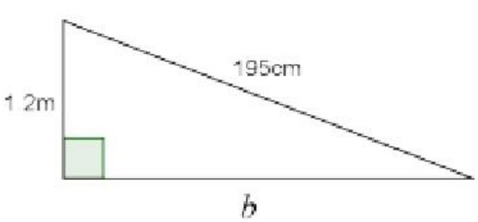


A)

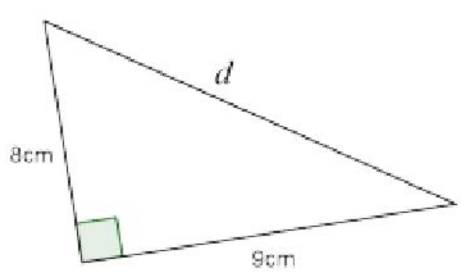


B)

Practice 2: Find the missing length using the Pythagorean Theorem.



A)

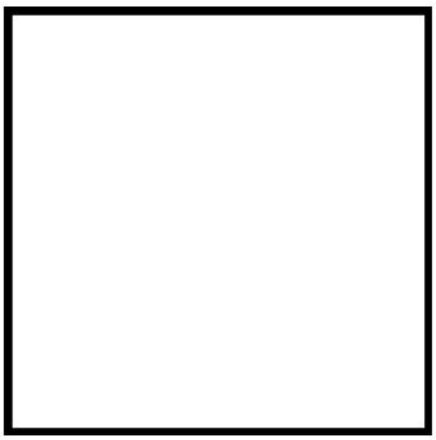


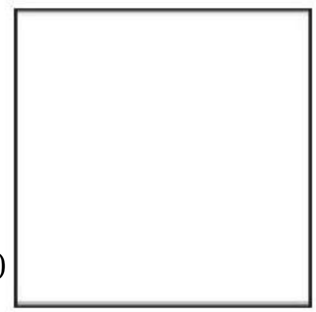
B)

Today, we are going to start working with radicals and right triangles.

Discovery Activity

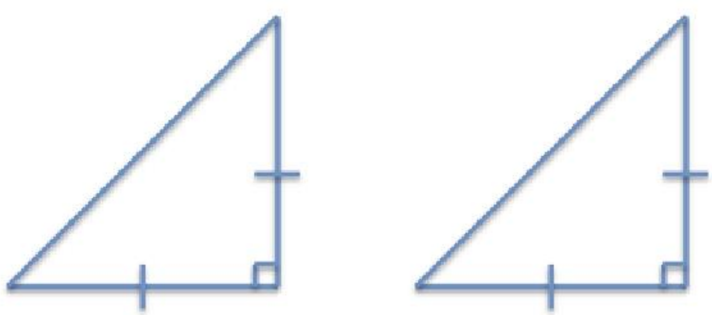
1) Measure the sides of the squares below.
2) Draw a diagonal from one corner to another one.
3) Using the Pythagorean Theorem, find the exact measure of the diagonal. Leave it as a radical if necessary.
4) Write and simplify the ratios from one side to another. 5) Answer the questions below.

A) 

B) 

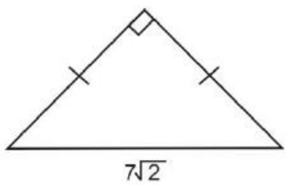
a) What is the measure of the angles in each triangle formed when bisected by the diagonal?
b) What is the ratio from one side to the other?
c) What is the ratio from one side to the hypotenuse?
d) What is the ratio from the hypotenuse to a side?
e) Were the ratios the same in both squares?

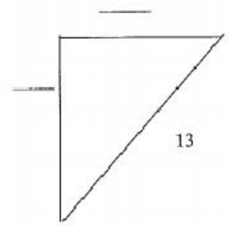
What we have just discovered is a special type of triangle called the ___________________, which happens to be an ____________________________ triangle. The ratios of the sides are always __________________, which, if we represented by "x", they would be ___________________. This information helps us as a shortcut in trying to find the length of a missing side or angle in this type of triangles. In other words:

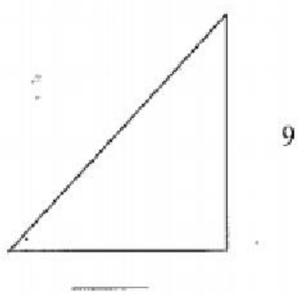


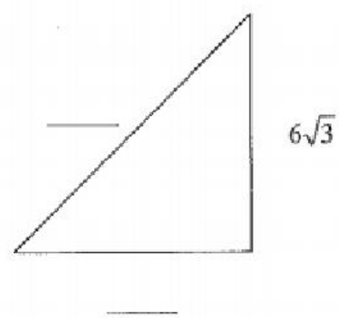
In this type of triangles, the hypotenuse will be across from the ________, and the legs will be across from the ________.

Example 3: Find the missing length in the following 45-45-90 triangles.

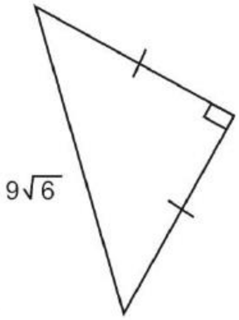
A) 

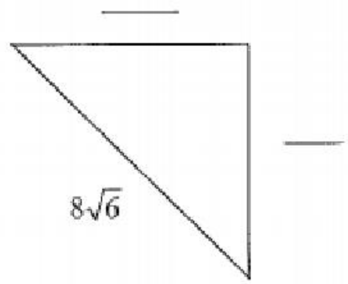
B) 

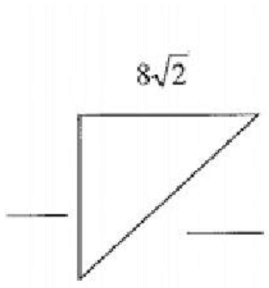
C) 

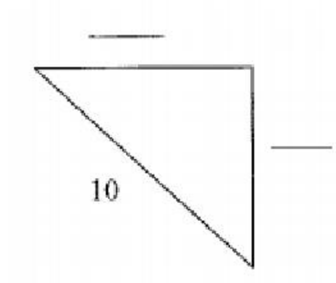
D) 

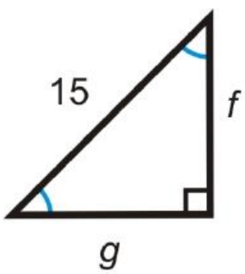
Practice 3: Find the missing length in the following 45-45-90 triangles.

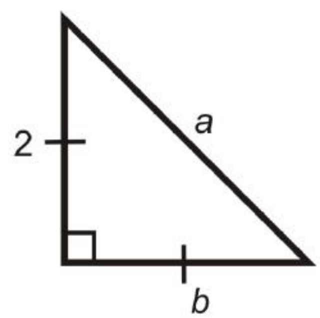
A) 

B) 

C) 

D) 

E) 

F) 

1) Construct an equilateral triangle with sides 2 inches long. Label the vertices *F*, *G* and *H*.

2) Find the midpoint of segment $\overline{FH}$ and label it *J*. Draw median $\overline{GJ}$.

3) Use a protractor to measure $\angle FGJ$, $\angle F$ and $\angle GJF$ and label them.

4) Use the Pythagorean Theorem to find *GJ*. Write it in simplest form. __________

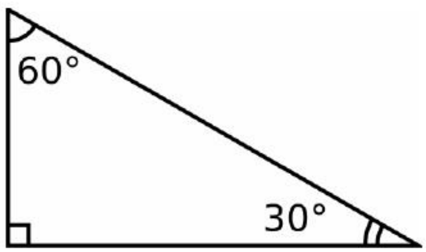
Repeat the activity for 4in (in the space below) and 5in (in the next page), and complete the table below.

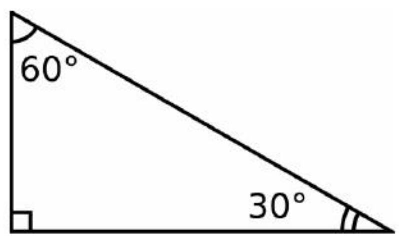
FG	*FJ*	*GJ*
2 in.		
4 in.		
5 in.		

Make a conjecture: What are the lengths of the long leg and the hypotenuse of a 30^o-60^o-90^o triangle with a short leg *n* units long?

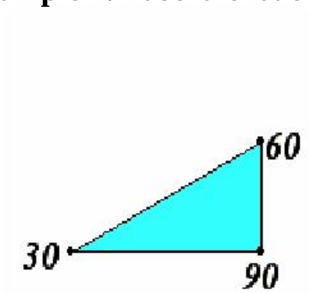
Objective: To find missing length of sides in special right triangles

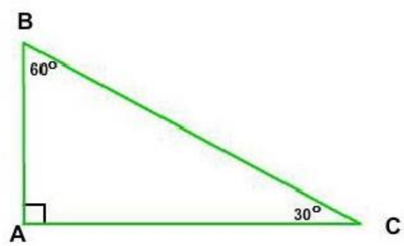
- Recall that 45-45-90 triangles always have a ratio of 1-1-$\sqrt{2}$ or x-x-$x\sqrt{2}$.
- Now, we are going to be working with the other kind of special right triangles that work similarly to 45-45-90 triangles.
- The angle measures of the other type of special right triangles are ___________________, and the ratios of the sides are ____________________. If we add an "*x*" to those ratios, the ratios become _______________________.
- In order to determine where the ratios go, we have to match small angle with small ratio, middle angle with middle ratio, and large angle with large ratio. In other words, the ____ is across from _______, _______ is across from ________, and _______ is across from __________.

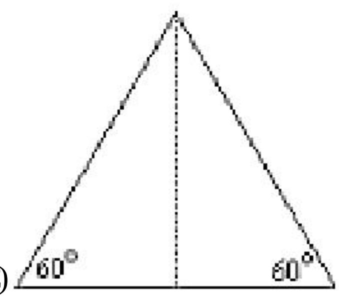




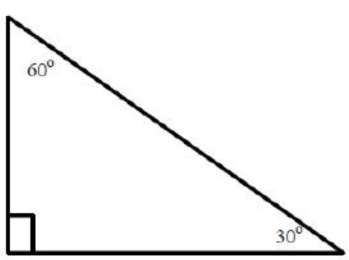
Example 1: Label the ratios of the sides of the 30-60-90 triangles below.

A) 

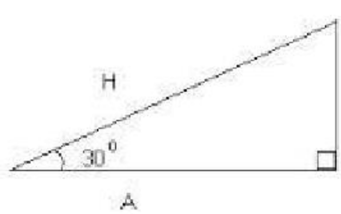
B) 

C) 

Practice 1: Label the ratios of the sides of the 30-60-90 triangles below.



A)



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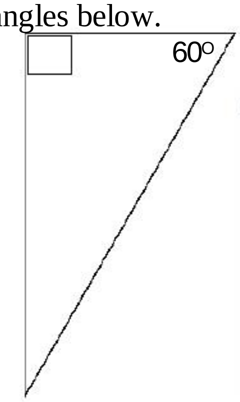

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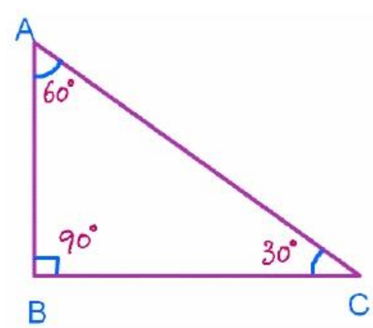
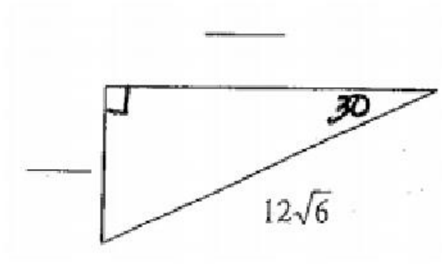


D)

When given the measure of one of the sides, we can use those ratios to find the other two lengths.

1) Label the ratios of the sides.
2) Set the measure given equal to the ratio that matches that side.
3) Solve for *x*.
4) Plug the value of *x* into the other two sides.

Example 2: Given the side measure of the 30-60-90 triangles below, find the length of the other two sides.

A)


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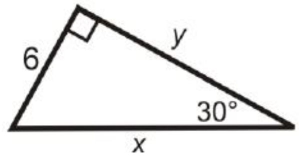

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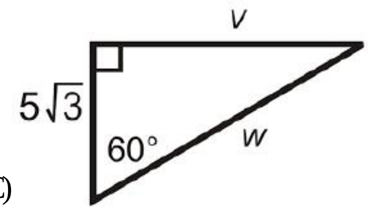

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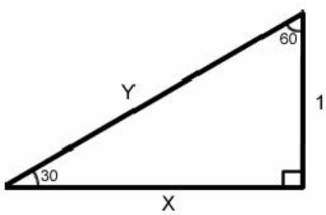

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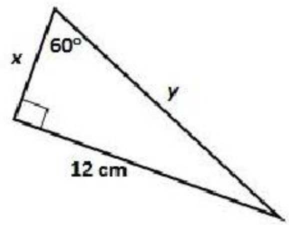
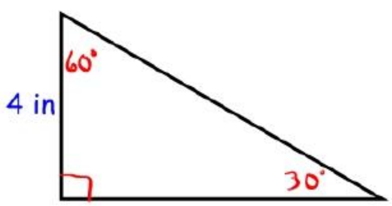


Practice 2: Given the side measure of the 30-60-90 triangles below, find the length of the other two sides.

A)


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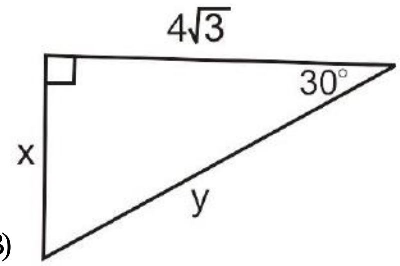

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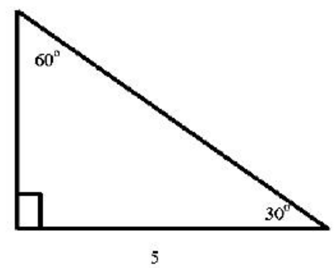

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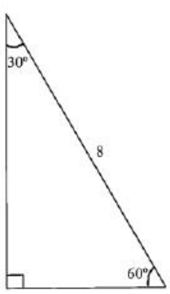

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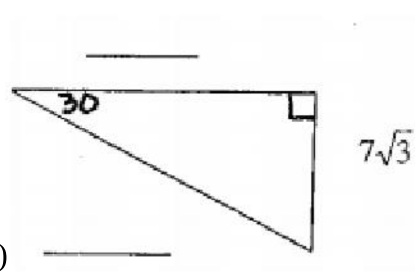

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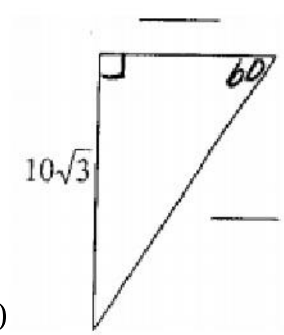
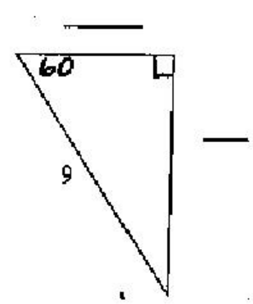


G)


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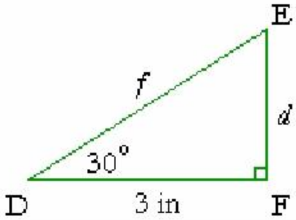

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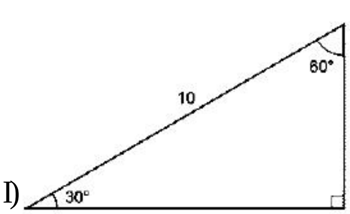

J)